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Relativistic psychometrics in subjective scaling

Natalia Yanova*

Department of Psychometrics and Educational Qualimetry, Altai State University, pr.Lenina 61, Barnaul 6560049, Russia
Monitoring Department, Psychology and Education Center, Komsomolsky Prospect 77, Barnaul 6560038, Russia

Abstract

The article announces the possibilities of semantic modeling in the development of feedback tools in social sciences. A new approach to the computational theory of perceptions (CTP) for analysis of mental object is proposed. The article demonstrates the implementation of relativistic psychometrics for the study of mental response (opinions, expectations and attitudes). The problem of image understanding and its significance is considered in combination of soft and hard computing. It is shown that the modeling of object (its coding and decoding in 'mental map') obeys the semiotic and mathematical logic. Computing with perceptions for the rules of mental representation proves their identity to the laws of conservation. The article demonstrates the versatility of the semiotic description of objects in Minkowski space. It also confirms by mathematical solution C. S. Peirce's metaphor, according to which the semiology of language is a truly universal algebra of relations.

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Keywords: mental representation; relativistic psychometrics; computational theory of perceptions; image understanding; significance; expert test system; psychosemiotics

1. Introduction

If computing with words[1] imitates the human way of thinking with linguistic information, so modeling with 'sign of words' imitates the human way of mental representation with semiotic information. Manipulation of visual perceptions is the brain's ability and manipulation of social perceptions is the mind's ability. But all mental games

*Natalia Yanova Tel. +7 -903-910-9081
E-mail address: yanova@sny-research.com

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controlled by the language games[2]. Because the language is a system of signs, the sign may be claimed as a dependent variable in all mental functions from emotion to cognition. ‘Sinn und Bedeutung’[3] are functional of sing. We use relativistic computing[4] for meaning and quantum computing[4] for sense. It’s the sign which provides encoding between the first and second signal systems. At the same time, the universal subject code[5,6] is responsible for categorization (semiotics of language), as the universal mental code is responsible for the conceptualization (semantics). This is how the formation of concepts, their signification and comprehension take place. All mental processes obey the principle of the sign. In this article we attempt to formalize the language of mental representation. We study semiotic process of thinking consistent with mental representation. We discuss a mathematical model of mental representation based on psychological and semiotic principles of perception and cognition. Why do people often see what is not there, and don't see what there is? The reasons for this trick are internal mental processes related to invariant sign operations in the scheme "sensation-perception-representation".

A transition of the sign from one system to another is no more than the encoding of its contents. Changing of the code modifies the interpretation of the object, not the object itself. The laws of its invariant transformations are constant. These are the ‘conservation laws’ of the object. They do not depend on the system of its representation and "work" in any of them. Their semiotics is universal and is of a meta-linguistic nature. These are universal constant ways of perceiving the world through objects and objects through words. In the study of mental response we are interested in semiotic focus to problem of ‘object world’[7]. We proceed from the hypothesis that the semiotic process of mental representation is universal and may be performed in semantic-mathematic model. This model formalizes new method of subjective scaling and explains semiotic solution in ‘progressing from perceptions to measurements’[8]. We suggest relativistic psychometric technology[9] and its software implementation. It is used in academic and applied research[10,11] as a test software module for expert systems (developer SNY-research Group[4]). The technology has been tested in education[12], sport, marketing, management, advertising, social politics. Universality of technology justified its research design for versatile application.

2. Semantic model of mental space

Let's specify the basic provisions of the measurement model ‘through property’.

2.1. Intensity and rigidity in property space

Classical approach to measurement performs description of any object through its properties. In vector form any object Ω of mental space reflects into vector *U* in a property space:

$$\vec{U} = \left\{ Q_1, Q_2, ..., Q_{j,} ..., Q_n \right\}$$ *, where coordinate Qj –the j-th property (j = 1, 2, .. n).* *(1)*

Bur the subjective perception “with property” includes two different concepts: the intensity and the rigidity. To distinguish them we have to introduce two components for property: *Vj*, for the intensity of property, *Uvj* for rigidness of property and U_H for object rigidity. The space, in which such a representation is realized, will be called semantic mental space. The properties in this space are presented with plane (multidimensional), but not single vector (one-dimensional). The object coordinates are angular (not linear) and called “the semantic coordinates’.

By adding the object coordinates, the properties do not change, unlike the rigidity (object’s mass).

Example of object definition: $\left\{ V_1, V_2, U_H \right\}$, i.e. $\Omega = \Omega' \rightarrow \left\{ V_1, V_2, U_H \right\}$,

Example of object’s sum:

$$\Omega_\Sigma = \Omega + \Omega' \rightarrow \left\{ V_1, V_2,\ 2 \cdot U_H \right\} \qquad (2)$$

Vector representation of objects becomes impossible, because in vector addition coordinates of properties are to be summed. To return to vector representation, let us associate vector $\vec{U}_j$ to each *j*-th quality in a certain space. It is evident that this vector has a certain correlation to intensity of *Vj* property.

2.2. From "liner" to "angular" representation

The way for the description of intensity of property is angle (cos φ_j). In this way may be used by a space of not less than two dimensions. Even when describing one property need a plane, where e1, e2 single vectors, $\vec{U}_\Sigma$ length of vector projection onto i-property plane $\vec{e}_j \times \vec{e}_H$, expressed through rigidiness U_j and intensity φ_j of property. The construction of semantic mental space is represented in Figure 1 and Figure 2.

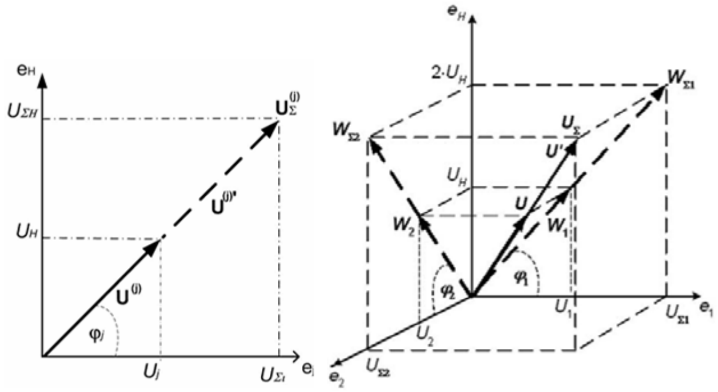

Fig.1. The plane (flat continuum) Fig.2. Semantic Coordinates

For two properties the mental space will be three-dimensional (e1, e2 , eH) .

Vector $\vec{U}_\Sigma = \vec{U}' + \vec{U}$ has semantic coordinates: $\vec{U}_\Sigma = \{U_{\Sigma 1}, U_{\Sigma 2}, 2 \cdot U_H\}$. Let us designate plane ($\vec{e}_j \times \vec{e}_H$) through Υ_j - the plane of the *j*-th property. Although, that coordinates $U_{\Sigma 1}$ and $U_{\Sigma 2}$ of vector $\vec{U}_\Sigma$ are twice as great as for vector $\vec{U}$ and $\vec{U}'$, the angle of inclination between projections of vectors $\vec{U}$ and $\vec{U}_\Sigma$ onto plane Υ_1 ($\vec{W}_1$ and $\vec{W}_{\Sigma 1}$ correspondingly) toward *e1* axis has remained intact. A similar statement can be repeated concerning angle φ_2,that characterizes the slope of $\vec{W}_2$ and $\vec{W}_{\Sigma 2}$ to *e2* axis onto the plane of the second characteristic (Υ_2).

2.3. Limits of semantic coordinates

Clarify the sense of semantic coordinates of *Uj*. Since $\varphi_j = \arccos\left(U_j / W_j \right)$, it would be convenient to juxtapose the cosines of angles (*j* =1, 2, …, *n*) to the corresponding property intensities *Vj*. Therefore, property intensities would be put as follows:

$$V_j = C_j \cdot \cos(\varphi_j) = C_j \cdot \left(U_j / W_j \right) = C_j \cdot \left(U_j \Big/ \sqrt{U_j^2 + (U_H)^2} \right), \tag{3}$$

where *Cj* - constants or scaling ratios that depend on the system of property units (*j* is a property index). Therefore, true characteristics will always be quantitatively restricted $|\cos\varphi| \le 1$. That is exactly why the scales for informant testing could be given by finite line segments with marked limits *Cj*.

2.4. The universal rule of 'property composition'

Since $V_j = C_j \cdot \cos\varphi_j$, then the angular coordinate for U_H in plane Υ_j would be determined as follows: $V_{Hj} = C_j \cdot \sin\varphi_j$ and, therefore, we can put down the identity:
$C_j^2 = V_{Hj}^2 + V_j^2$ or $V_{Hj}^2 = C_j^2 - V_j^2$.

By dividing the latter identity by V_{Hj}^2 we receive: $1 = \left(\frac{C_j}{V_{Hj}}\right) - \left(\frac{V_j}{V_{Hj}}\right)$. To simplify further calculations, put the last identity as: $1 = \mathrm{ch}^2\,\varphi_j - \mathrm{sh}^2\,\varphi_j$, where $\mathrm{ch}\,\varphi_j = \frac{C_j}{V_{Hj}}$ and $\mathrm{sh}\,\varphi_j = \frac{V_j}{V_{Hj}}$ - are hyperbolic cosines and sinus respectively.

Since $\mathrm{th}\,\varphi_j = \frac{\mathrm{sh}\,\varphi_j}{\mathrm{ch}\,\varphi_j} = \frac{V_j / V_{Hj}}{C_j / V_{Hj}} = \frac{V_j}{C_j}$, then the rule for the composition of two intensities of the *j*-th property (V_{j_1} and V_{j_2}) would be determined by the formula for the tangent of two angles summed. Indeed, having represented a hyperbolic tangent as a sum of two angles, we would have: $\frac{V_j}{C_j} = \mathrm{th}\,\varphi_j = \mathrm{th}\left(\varphi_{j_1} + \varphi_{j_2}\right) = \frac{\mathrm{th}\,\varphi_{j_1} + \mathrm{th}\,\varphi_{j_2}}{1 + \mathrm{th}\,\varphi_{j_1} \cdot \mathrm{th}\,\varphi_{j_2}} = \frac{\left(V_{j_1}/C_j\right) + \left(V_{j_2}/C_j\right)}{1 + V_{j_1} \cdot V_{j_2} / C_j^2}$. Therefore, canceling *Cj*, we receive:

$$V_j = \frac{V_{j_1} + V_{j_2}}{1 + \frac{V_{j_1} \cdot V_{j_2}}{C_j^2}} \qquad \frac{C_j + C_j}{1 + \frac{C_j \cdot C_j}{C_j^2}} = \frac{2C_j}{2} = C_j \tag{4}$$

This general rule of property composition (4) completely coincides with the rule of velocity composition in relativist mechanics and is realized in Minkovsky space[13]. Eq. (4) assumes a classical form: $V_j = V_{j_1} + V_{j_2}$.

However, while most problems in physics are solved with the use of a classical setting, in sociology and psychology consideration of non-linearity is a must, due to a lower rigidity of the properties studied. Even an ordinary calculation of mean values can lead to grave errors. It is easy to test that even the sum of limiting values *Cj* still yields a limiting value (5). Thus, restriction by limiting values $\left(\pm C_j\right)$ of all properties within the limits of their intensity is a mere effect of a definite way of object representation on the mental presentation.

2.5. Calculation of rigidity

Let us now clarify the sense of V_{Hj} . Basing on Eq. (1) and Eq. (2), we get the formula (5):

$$V_{H_j}{}^2 = C_j^2 - V_j^2 = C_j^2 - \left(\frac{V_{j_1} + V_{j_2}}{1 + \frac{V_{j_1} \cdot V_{j_2}}{C_j^2}}\right)^2 = \frac{\left(C_j - V_{j_1}\right)^2 - V_{j_2}^2\left(1 - \frac{V_{j_1}^2}{C_j^2}\right)}{\left(1 + \frac{V_{j_1} \cdot V_{j_2}}{C_j^2}\right)^2} = \frac{V_{H_{j_1}}^2 \cdot V_{H_{j_2}}^2}{C_j^2\left(1 + \frac{V_{j_1} \cdot V_{j_2}}{C_j^2}\right)^2}$$

$$V_{H_j} = \frac{V_{H_{j_1}} \cdot V_{H_{j_2}}}{C_j\left(1 + \frac{V_{j_1} \cdot V_{j_2}}{C_j^2}\right)} = \frac{V_{H_{j_1}} \cdot \sqrt{1 - \frac{V_{j_2}^2}{C_j^2}}}{\left(1 + \frac{V_{j_1} \cdot V_{j_2}}{C_j^2}\right)}$$

2.6. Normalization of semantic coordinates.

Formula for rigidity U_H and semantic coordinates of objects U_j in a general case:

$$\operatorname{ctg}(\varphi_j) = U_j / U_H = v_j / v_{Hj} = u_j / u_H \ , \tag{6}$$

where u_j и u_H are normalized semantic coordinates, and v_j are normalized property intensities:
$\vec{u} = \vec{U} / |\vec{U}|$, $v_{Hj} = V_{Hj} / C_j = \sin\varphi_j$, $v_j = V_j / C_j = \cos\varphi_j$,

It issues from Eq. (6) that: $U_j = U_H \cdot (v_j / v_{Hj})$. *(7)*

Since, $\vec{\mathrm{U}}^2 = \vec{\mathrm{U}}'^2 + \vec{\mathrm{U}}_H^2 = \sum_{j=1}^{n} U_j'^2 + U_H^2 = \sum_{j=1}^{n} U_H^2 \cdot \frac{v_j^2}{v_{Hj}^2} + U_H^2 = U_H^2 \cdot \left[\left(\sum_{j=1}^{n} \frac{v_i^2}{v_{Hj}^2} \right) + 1 \right] = U_H^2 \cdot \left[\left(\sum_{j=1}^{n} \frac{v_j^2}{1 - v_j^2} \right) + 1 \right],$

Then $$U_H = \frac{|\vec{\mathrm{U}}|}{\sqrt{\sum_{j=1}^{n} \left(\frac{v_i^2}{1 - v_j^2} \right) + 1}} \tag{8}$$

Here: $\vec{\mathrm{U}}' = \{U_1, U_2, ..., U_n, 0\}$; $\vec{\mathrm{U}}_\mathrm{H} = \{0, 0, ..., 0, U_H\}$. It issues from Eq. (7) and Eq. (8) that:

$$u_H = \frac{1}{\sqrt{\sum_{j=1}^{n} \left(\frac{v_j^2}{1 - v_j^2} \right) + 1}}; \tag{9}$$

$$u_j = u_H \cdot \frac{v_j}{v_{Hj}} = u_H \cdot \frac{v_j}{\sqrt{1 - v_j^2}} = u_H \cdot \frac{V_j / C_j}{\sqrt{1 - \frac{V_j^2}{C_j^2}}}; \tag{10}$$

Eq. (9) is used for calculation of normed object rigidity as a whole, while Eq. (10) for calculation of normed semantic coordinates. Then absolute semantic coordinates U could be obtained through the formula:

$$U_j = \frac{V_j \cdot \left(u_H \cdot |\vec{\mathrm{U}}| / C_j \right)}{\sqrt{1 - \frac{V_j^2}{C_j^2}}} = \frac{V_j \cdot m_{0j}}{\sqrt{1 - \frac{V_j^2}{C_j^2}}} = m_j V_j \tag{11}$$

where m_{0j} is «zero» rigidity $m_{0j} = u_H \cdot |\vec{\mathrm{U}}| / C_j$ and m_j - full rigidity: $m_j = \frac{m_{0j}}{\sqrt{1 - \frac{V_j^2}{C_j^2}}}$, (12)

In physics it could be put down as follows: $P_j = m_j \cdot V_j$; where Pj - impulse, V j – velocity, m_j - object mass.

It is clear from Eq.(12) that full rigidity with the rising intensity grows in a non-linear. In fact, it means that an absolute limit of any property is a thing unattainable. Therefore, semantic coordinates are similar to what physicists call impulse representation. Semantic model of mental representation proved that semantic description of a closed system of objects is retained in any instances of its transformation. It enables to apply laws of conservation in the humanities, which makes their prediction potential equivalent to that of the natural sciences.

2.7. Universal Invariants.

The change of semantic coordinate u_ρ from 0 to U_ρ transforms the vector that determines an object, from state $\vec{U}_0$ to state $\vec{U}$. In this connection, projections of semantic vector onto the plane $u_H \times u_\nu$ axis must remain unaltered under consistent changes of U_ρ, which is equivalent to the change in only one property of object. Let us express the above alterations in terms of properties: $\Delta\vec{U} = \vec{U} - \vec{U}_0 = \vec{W}_\rho - \vec{U}_H = \vec{U}_\rho$

$$U_\rho^2 = \left(\vec{W}_\rho - \vec{U}_H\right)^2 = \vec{W}_\rho^2 - 2\cdot\left|\vec{W}_\rho\right|\cdot\left|\vec{U}_H\right|\cos\left(\frac{\pi}{2}-\varphi\right) + \vec{U}_H^2 =$$
$$= \left(\frac{U_H}{\sin\varphi}\right)^2 - 2\left(\frac{U_H}{\sin\varphi}\right)U_H\sin\varphi + U_H^2 = \left(\frac{U_H}{\sin\varphi}\right)^2 - U_H^2 = \left(\frac{U_H}{\sqrt{1-\cos^2\varphi}}\right)^2 - U_H^2 = \qquad (13)$$
$$= \frac{1}{C^2}\left(\frac{\frac{U_H}{C}C^2}{\sqrt{1-\frac{V^2}{C^2}}}\right)^2 - \frac{1}{C^2}\left(\frac{U_H}{C}C^2\right)^2 = \frac{1}{C^2}\left[\left(\frac{m_o C^2}{\sqrt{1-\frac{V^2}{C^2}}}\right)^2 - \left(m_o C^2\right)^2\right] = \left(\frac{E^2}{C^2} - \frac{E_0^2}{C^2}\right)$$

Where

$$E = \frac{m_o C^2}{\sqrt{1-\frac{V^2}{C^2}}} = mC^2; \quad E_0 = m_o C^2 \qquad (14)$$

Thus, we have obtained:

$$U_\rho^2 = P^2 = \left(\frac{E^2}{C^2} - m_o^2 C^2\right) \quad \text{or} \quad \frac{E^2}{C^2} = P^2 + m_o^2 C^2 \qquad (15)$$

Eq. (14) is known in physics as the relativist energy-impulse relation, while Eq. (12) reflect the relations between energy and mass (rigidity). It should be noted that, object mass is not an equipotent property, but a summary characteristic of properties' rigidity, being defined through several independent values: $m = \sqrt{U_H^2 + \sum_i U_i^2}$

It issues, that full mass, unlike properties, is not limited, that is why all values depending on it, are not properties. The laws of impulse and energy conservation are adequate to conservation of "semantic definition" of object.

3. Experimental realization of method

The measurement procedure is realized by means of software solution and partly resembles the repertory grid technique[14]. This method of subjective scaling use MDS-paradigm[15] as a research design and includes different solutions for scale construction (metric, non-metric, replicated, weighted). Psychometrics school employs new algorithm (IRT) according to the Thurstone's[16,17] method or the Young&Householder's[18,19,20] method. In our case, a relativistic scale is used. Its algorithm imitates the process of subjective estimation with rank and interval procedure of scaling. The calculation of interval length is performed in the shown relativistic mathematical model. Relativistic scale is not a linear scale of "defined" intervals with equal distances between its marks. It is a relative scale in which values are not set, but calculated and "compressed" to the ultimate limit. The program calculates the value of the scale interval in a subsequent step in relation to the value of the scale interval in a previous step. This is done by calculating the scale "compression" coefficient. Calculation of the scale coefficient 'k' is shown in Fig. 3. E. g., the scale coefficient in the second step is calculated by the formula: $k_2 = \frac{V_2^{(1)} - V_1^{(1)}}{V_2^{(2)} - V_1^{(2)}}$.

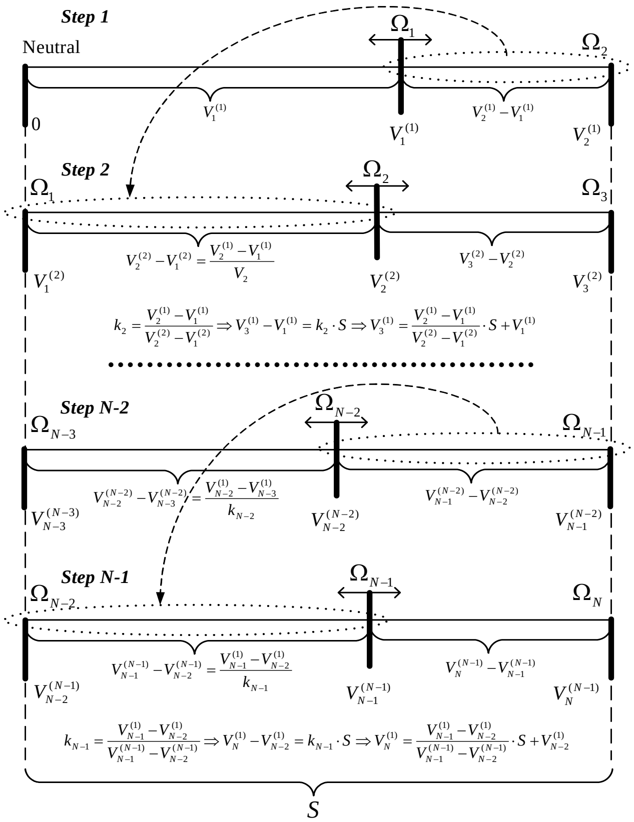

Fig.3. Measurement algorithm in relativistic scale

Where the lower index is the number of the object, and the upper index is the number of the step where the measurement is performed. At each step the scale has its own scale coefficient (size of compression). Since

$$V_3^{(1)} - V_1^{(1)} = k_2\left(V_3^{(2)} - V_1^{(2)}\right) = k_2 \cdot S$$

the assessment of the third object in scale size of the first step will be calculated by the formula: $V_3^{(1)} = k_2 \cdot S + V_1^{(1)}$, where S is the length of the slider. Using the recurrence relation:

$$V_i^{(1)} = \frac{V_{i-1}^{(1)} - V_{i-2}^{(1)}}{V_{i-1}^{(i)} - V_{i-2}^{(i)}} \cdot S + V_{i-2}^{(1)}, \quad (i = 2, 3, ..., n-1) \qquad (16)$$

we carry out the recalculation of the property intensity assessment of the 'i' object by bringing the scale size of (i-1) step to the scale size of first step. Thus, we carry out the transition from one reference system to another (dashed arrow in Fig. 3).Then everything is repeated until the assessment Ω_{N-1} in the (N-1) step. Quantitative assessment of the last object in a list (Ω_N) is obtained automatically. In this algorithm we are continuously recalculate the scale size at every step. Because of this, we can construct a continuous scale of any length, using a constant "length" of the screen slider (S) at each step of the measurement procedure.

4. Conclusion and future directions

We are expanding the boundaries of the CTP paradigm in the study of image significance. Due to the fact that the meaning is a function of the sign, not the word, so we move on from the words of the speech to the sign of the language. CWW method tries to translate the word of perception in digit format. We try to translate any sign of perception in digit format. Both methods require accurate measurement (at output) for "inaccurate words" (at input).

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